Competition and Collaboration in Wireless Networks

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Research Trends in Wireless Nets

• The Past Two Decades: Key Developments at the Link Level
  – MIMO
  – MUD
  – Turbo

• Today: An Increased Focus on Interactions Among Nodes
  – Competition
    • Cognitive radio
    • Information theoretic security
    • Game theoretic modeling, analysis & design
  – Collaboration
    • Network coding
    • Cooperative transmission & relaying
    • Multi-hop transmission & coalition games
    • Collaborative beam-forming
    • Collaborative inference
Today’s Talk - Two Parts

- **Energy Games**: Competition in Multiple Access Communication Networks
- **Distributed Inference**: Collaboration in Wireless Sensor Networks (WSNs)
ENERGY GAMES: COMPETITION IN THE MAC

[Joint work with Farhad Meshkati, Stuart Schwartz, et al.]
Energy Games

- Terminals transmit to an access point via a multiple-access channel.
- Users are like players in a game, competing for resources to transmit their data to the AP.
- The action of each user affects the others.
- We can model this as a competitive game, with payoff measured in bits-per-joule.

Competition in MA Communication Networks
Game Theoretic Framework


Game: \( G = [\{1, \ldots, K\}, \{A_k\}, \{u_k\}] \)

\( K \): total number of terminals  
\( A_k \): set of strategies for terminal \( k \)  
\( u_k \): utility function for terminal \( k \)

\( u_k = \text{utility} = \frac{\text{throughput}}{\text{transmit power}} = \frac{T_k}{p_k} \left[ \frac{\text{bits}}{\text{Joule}} \right] \)

\( T_k = R_k f(\gamma_k) \), where \( f(\gamma_k) \) is the frame success rate, and \( \gamma_k \) is the received SINR of user \( k \).

Competition in MA Communication Networks
An Uplink Game

• For a fixed linear MUD at the uplink receiver, each user selects its transmit power to maximize its own utility.

• Th’m [w/ Mandayam, T-COM 05]: $f$ sigmoidal $\Rightarrow$ Nash equilibrium (i.e., no user can unilaterally improve its utility) is reached when each user chooses a transmit power that achieves $\gamma^*$:

$$f(\gamma^*) = \gamma^* f'(\gamma^*)$$

• I.e., Nash equilibrium (NE) requires SINR balancing.
Remarks

• The NE is unique, and can be reached iteratively as the unique fixed point of a nonlinear map.

• The NE as an Analytical Tool:
  – We can use the NE to examine the effects on energy efficiency of various network design choices.
  – E.g., we can compare receiver choices: the matched filter, (zero-forcing) decorrelator, and MMSE detector.

*Competition in MA Communication Networks*
User 1: 010...
User 2: 110...
User K: 011...

Channel Gains: \( \{h_{k,p}\} \)
Random CDMA: $K$ terminals; spreading gain $N$

Load: $\alpha = K/N$ (i.e., the number of users per dimension)

Large-system limit: $K, N \to \infty$, with $\alpha$ fixed.

*Competition in MA Communication Networks*
Social Optimality

• The Pareto (or socially) optimal solution, chooses the transmit power so that no user’s utility can be improved without decreasing that of another.

• The Pareto solution is generally hard to find.

• The Nash equilibrium solution not generally Pareto optimal.

• But, it’s close.

Competition in MA Communication Networks
Example: Nash & Pareto Optima

Utility vs. Load

Competition in MA Communication Networks
Effects of Delay QoS

[w/ FM & SC, submitted to Trans. IT]

• For some traffic, delay is a key element of service quality.

• Delay model (ARQ):
  - $X$ represents the number of transmissions needed for a given packet to be received without error, so that:
    \[
    P(X=m) = f(\gamma) \left[1 - f(\gamma)\right]^{m-1}, \ m = 0, 1, \ldots
    \]
  - We can represent a delay requirement as a pair $(D, \beta)$:
    \[
    P(X \leq D) \geq \beta \iff \gamma \geq \gamma' \]
  - Thus, we have a constrained game, with $\gamma_k \geq \gamma'_k$.

Competition in MA Communication Networks
NE for Multiple Delay Classes

- Traffic is typically heterogeneous with multiple delay classes.
- A given delay class $c$ will have its own SINR constraint: $\gamma_c'$.
- At NE all users in class $c$ will SINR-balance to $\max\{\gamma^*, \gamma_c'\}$.
- Tight delay constraints on one class can affect the energy efficiencies of all traffic due to increased interference levels.
2-Class Example: Utility Loss

- RCDMA in the large-system limit: $K, N \to \infty$, with $\alpha = K/N$ fixed.
- Class A: $(D_A, \beta_A) = (1, 0.99)$
- Class B: $(D_B, \beta_B) = (3, 0.90)$

Competition in MA Communication Networks
Finite Backlog Case

[w/ R. Balan, T-COM, to appear.]

- Poisson packet arrivals
- FIFO’ed packets transmitted via ARQ
- **QoS**: *(source rate, ave. delay)*
- Translates into a lower bound on SINR
- Constrained Nash game (on transmit power & rate)
- Leads to “size” of a user quantifying the resources required to deliver QoS.
- NE exists only when the sum of the users “sizes” is < 1.

*Competition in MA Communication Networks*
Utility is normalized by $B \times SNR$, and the normalized delay is $D \times B$. The combined “size” of the other users is 0.2.

*Competition in MA Communication Networks*
Enhancements

- **Nonlinear MUD (ML, MAP, PIC, etc.):** Results apply to nonlinear MUD for RCDMA in the large system limit. [w/ D. Guo, FM & SC; T-WC, to appear]

- **Multicarrier CDMA:** Actions include choice of a carrier. [w/ M. Chiang, FM & SC; JSAC’06]

- **UWB:** Rich scatter. [w/ G. Bacci, M. Luise & A. Tulino: JSTSP, to appear]

- **Adaptive Modulation:** Actions include choice of a modulation index [w/ A. Goldsmith, FM & SC: JSAC’07] or waveform [w/ S. Buzzi; EWC’07].

*Competition in MA Communication Networks*
COLLABORATIVE INFEERENCE IN WSNs

[Joint work with Sanj Kulkarni, Joel Predd, et al.]
Salient features of WSNs:

- The primary application is inference
- Information at different terminals is often correlated
- Energy is often severely limited

Collaborative inference:

- Sensors work together to make inferences, while conserving resources (i.e., “bandwidth & batteries”)
- Here, we’ll examine collaborative learning
Classical (Supervised) Learning

- Input space $X = \mathbb{R}^d$; output space $Y = \mathbb{R}$

- $(X,Y)$ is an $X \times Y$-valued r.v. with $(X,Y) \sim P_{XY}$

- Design $f: X \rightarrow Y$ to predict outputs from inputs and minimize expected loss; e.g., $\mathbb{E}\{|f(X)-Y|^2\}$

- $P_{XY}$ is unknown

- So, construct $f$ from examples: $S = \{(x_i, y_i)\}_{i=1}^n$

Collaborative Inference
A Model for Dist’d’d Learning in WSNs

“A distributed sampling device with a wireless interface”

- Sensor $i$ measures $S_i = \{(x_j, y_j)\} \subseteq S$.

- This division defines a topology, which in turn shapes the nature of collaboration.

Collaborative Inference
A Centralized Approach

- Sensor $i$ sends $S_i = \{(x_j, y_j)\}$ to a centralized processor.

- “Learn” using (reproducing) kernel methods:
  - For a positive semi-definite kernel $K(,)$:
    \[
    f_* = \arg \min_{f \in \mathcal{H}_K} \sum_{i=1}^{n} (f(x_j) - y_i)^2 + \lambda \|f\|_{\mathcal{H}_K}^2
    \]

- **Assumption**: energy and bandwidth constraints preclude the sensors from sending $S_i = \{(x_j, y_j)\} \subseteq S$ for centralized processing.

**Collaborative Inference**
The Seed of a Model ...

- Sensor $i$ measures $S_i = \{(x_j, y_j)\} \subseteq S$

**Assumption**: Sensor $i$ can access all neighboring sensors’ measured data.

- **Informal justification**: local communication is efficient

**Collaborative Inference**
A General Model

- \( m \) learning agents (i.e., sensors)
- \( n \) training examples

\[ S = \{(x_i, y_i)\}_{i=1}^{n} \]

Collaborative Inference
Example:
Centralized Learning

Collaborative Inference
Example: Spatio-Temporal Field Estimation

Collaborative Inference
Example
A Public Database

Collaborative Inference
The General Case

- m **learning agents** (i.e., sensors)
- n **training examples**

\[ S = \{(x_i, y_i)\}_{i=1}^n \]

Collaborative Inference
“Local” Learning
A Natural Approach

\[ \hat{f}_1 = \arg\min_{f \in \mathcal{H}_k} \frac{1}{|N_1|} \sum_{j \in N_1} (f(x_j) - y_j)^2 + \lambda_1 \|f\|^2_{\mathcal{H}_k} \]

\[ \hat{f}_m = \arg\min_{f \in \mathcal{H}_k} \frac{1}{|N_m|} \sum_{j \in N_m} (f(x_j) - y_j)^2 + \lambda_m \|f\|^2_{\mathcal{H}_k} \]

Collaborative Inference
Local Learning is “Locally Incoherent”

Local incoherence:

Sensor 1 and sensor m both train with \((x_1, y_1)\) but \(\hat{f}_1(x_1) \neq \hat{f}_m(x_1)\)
• “Local learning” requires only local communication.

• However, it leads to local incoherence, which is (provably) “undesirable”.

• Can agents (i.e., sensors) collaborate to gain the “optimality” of coherence, while retaining the efficiency of locality?

Collaborative Inference
A Collaborative Training Algorithm

Intuition

- Use local learning as a building block.

Iterate over sensors $s = 1, \ldots, m$

Sensor $s$

Computes using local data:

$$\hat{f}_s = \arg \min_{f \in \mathcal{H}_K} \sum_{j \in N_s} (f(x_j) - y_j)^2 + \lambda_s \|f\|_{\mathcal{H}_K}^2$$

Updates labels of local data:

$$\{x_j, \hat{f}_s(x_j)\}_{j \in N_s} \rightarrow \{(x_j, y_j)\}_{j \in N_s}$$

end

Collaborative Regression
A Collaborative Training Algorithm

Intuition (cont’d)

• Need multiple passes + inertia term

Initialize: \( \hat{f}_{s,0} = 0 \in \mathcal{H}_K \)

for \( t=1, \ldots, T \)

Iterate over sensors \( s = 1, \ldots, m \)

sensor \( s \)

Computes using local data:

\[
\hat{f}_{s,t} = \arg \min_{f \in \mathcal{H}_K} \sum_{j \in N_s} (f(x_j) - y_j)^2 + \lambda_s \| f - f_{s,t-1} \|_{\mathcal{H}_K}^2
\]

Updates labels of local data:

\[
\{x_j, \hat{f}_{s,t}(x_j)\}_{j \in N_s} \rightarrow \{(x_j, y_j)\}_{j \in N_s}
\]

Collaborative Regression
To initialize, the sensors:
- agree on a kernel $K(.,.)$
- localize (i.e., estimate $x_i$)
- share positions with neighbors
- measure field locally (i.e. observe $y_i$)
- set $z_i = y_i$

To estimate the field:
for $t=1,...,T$
  for $s = 1,..., N$
    Query: Sensor $s$ queries $z_i$ from neighbors
    Compute: $f_{s,t} = \arg\min_{f \in \mathcal{F}} \sum_{j \in N_s} (f(x_j) - z_{j,t-1})^2 + \lambda_s \| f - f_{s,t-1} \|_f^2$
    Update: Updates neighbors $z_i = f_{s,t}(x_i)$

Collaborative Inference
A Collaborative Algorithm (Cont’d)

\[ f_{1,t} = \arg \min_{f \in \mathcal{H}_k} \sum_{j \in \{1,2,6\}} (f(x_j) - y_j)^2 + \lambda_1 \| f - f_{1,t-1} \|^2_{\mathcal{H}_k} \]

Collaborative Inference

\[ f_{4,t} = \arg \min_{f \in \mathcal{H}_k} \sum_{j \in \{3,5,7\}} (f(x_j) - y_j)^2 + \lambda_4 \| f - f_{4,t-1} \|^2_{\mathcal{H}_k} \]
**Properties of the Algorithm**

1. $\hat{f}_{s,T}$ converges (in norm) to a relaxation of centralized estimate as $T \to \infty$.

2. $\left\{ \lim_{T \to \infty} \hat{f}_{s,T} \right\}_{s=1}^{m}$ is locally coherent and satisfies

   \[
   \lim_{T \to \infty} \hat{f}_{s,T} \in \text{span}\{K(\cdot, x_j)\}_{j \in N_s}
   \]

3. $\left\{ \hat{f}_{s,t} \right\}_{s=1}^{m}$ improves with every update.

4. Within a connectivity assumption, with i.i.d. exemplars, and appropriate behavior of the $\lambda_s$'s, the local estimates converge (in RKHS norm) to the conditional mean as $n \to \infty$.

**Collaborative Inference**
Experiments

• \( n = 50 \) sensors uniformly distributed about \([-1, 1]\)

• Sensor \( i \) observes \( y_i = f(x_i) + n_i \)
  - \( \{n_i\} \) is i.i.d. standard normal
  - regression function \( f \) is linear (Case 1) or sinusoidal (Case 2)

• Sensors \( i \) and \( j \) are neighbors iff \( |x_i - x_j| < r \)

• Sensors employ linear (Case 1) or Gaussian (Case 2) kernel
How Does Collaboration Affect Globalization Error?

Connectivity vs. Error rate

Case 1

Case 1, n=50, S=300

Case 2

Case 2, n=50, S=300

Test Error

Collaborative Inference
• Overall error decreases with size of the neighborhoods.
• But, energy consumed by message-passing increases with neighborhood size.
• Question: What are the trade-offs?
Mean-Square Error vs. $n$

$r_N = n^\alpha$

$\alpha = \{0.30, 0.35, 0.40, 0.45\}$

PKP

Centralized

Local Averaging

Collaborative Inference

$n$ (number of sensors)
Energy-per-Sensor vs. $n$

$r_N = n^{\alpha}$

$\alpha = \{0.30, 0.35, 0.40, 0.45\}$

PKP
Centralized
Local Averaging

Collaborative Inference
**Related Results**

- **Consistency w. Limited Capacity**
  
  \[ w/ \text{SK} \& \text{JP}, \text{IT} 06 \]

- **Collaborative Beamforming**
  
  \[ w/ \text{Mitran, Ochiai} \& \text{Tarokh}, \text{SP 05} \]

- **Judgment Aggregation**
  
  \[ w/ \text{Osherson, SK} \& \text{JP, preprint} \]

**Collaborative Inference**

![Diagram showing signal transmission and reception with multiple access points and user devices, illustrating the concept of collaborative inference.](Image)
Summary

- **Energy Games:**
  
  Characterization of Energy Efficiency Via *Nash Equilibrium*

- **Distributed Inference:**
  
  Collaboration Via *Message Passing*
Thank You!